Hamiltonian in 5-Connected Graph

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Abstract— One of the branches of mathematics that studies the properties of graphs is graph theory. The purpose of this study is to find out how to prove the Hamiltonian on a 5-connected graph. Through the stages, including modeling a complete graph, as well as modeling a 5-connected graph using vertices and cut edges, it was found that K_6 is a graph that satisfies the characteristics of a 5-connected graph. Analysis of Hamiltonian on 5-connected graph that 5-connected graph is Hamilton's invention, but it is not uniquely Hamiltonian because it has more than one Hamiltonian circuit.

Keywords— k-connected graph; Hamiltonian; vertex cutset; edge cutset.

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I. INTRODUCTION

Graphs can be used to represent objects in Discrete Mathematics with some conditions. Along with the development of time, research on graph theory is growing rapidly [1]. In the application of graphs, applied modeling can be used to facilitate problem analysis. In everyday life, you can also find the application of graphs, such as the research conducted regarding the application of graphs to find the shortest route for transporting waste using the Hamilton approach[2]. A graph can contain Hamilton paths and Hamilton circuits, it can also not contain both, and or contain one of them. If a graph has a Hamiltonian circuit, it is called a Hamiltonian. The existence of Hamilton paths, and Hamilton circuits that focus on 4-connected graphs are concluded that 4-connected graphs are not Uniquely Hamiltonian [3]. Based on the background description above, as a form of development from previous research on Hamiltonian trajectories in 4-connected graphs, the researcher intends to analyze the Hamiltonian in 5-Connected graphs with the title "Hamiltonian in 5-Connected Graphs". This study aims to prove the existence of Hamiltonian circuits, as well as Hamiltonian in 5-Connected

II. RESEARCH METHOD

The type of research used in this thesis is a literature review. In conducting research, researchers collect data or sources related to the research topic. This method is called a literature study [4]. Researchers study articles, and other sources related to the research title. The literature study will then produce a theoretical basis for determining whether
Here are the research steps:

a. Conducting literature study on k-connected graph and Hamiltonian.
b. Represented a graph G is a complete graph according to the theorem and definition.
c. An example of a synchronous graph G is defined with the definition of a k-connected graph, where k=5.
d. Prove the 5-connected graph according to the definition of Hamiltonian.

III. RESULT AND DISCUSSION

A. Forming 5-connected graph

The initial stage in this study is to select a graph G that meets the definition of a complete graph, starting from a graph with three vertices. This limitation is intended to facilitate the determination of the desired graph G model. The next step is that the selected graph G must be able to form a 5-connected graph.

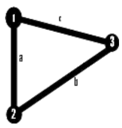
A one-vertex graph forms an isolated vertex where the graph has no edges. The isolated vertex graph can be seen in Figure 1.

Figure 1. An isolated vertex graph

A graph with two vertices can form a connected graph but does not form a k-connected graph. The graph of two vertices can be seen in Figure 2.

Figure 2. Graph of 2 vertices

When $n = 3$ where n is the number of vertices, and according to the **definition 1** complete graph (K_n) is a simple graph with n vertices where each pair of vertices is connected by an edge [5] and

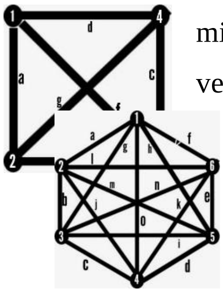


theorem 1 the number of sides in *complete graph* dengan $n \geq 3$ can be determined by $n \times \frac{1}{2}(n-1)$. The graph that is formed is as shown in Figure 3.

Figure 3. K_3 graph

Because Figure 3 fulfills the initial stage, the graph in Figure 3 can be called K_3 graph The next step is to form the graph K_3 into a k-connected graph.

Definition 2



k-connected graph is the minimum number of omissions from the combination of two cutset vertices and cutset edges as vertices and intersections of a connected graph to become an unconnected graph where $k(G) = k'G = k$.

Theorem 2

Complete graph K_n, $k(K_n) = n - 1$

Theorem 3

A graph G is called a k-connected graph if $k(G) = k$

k-connected graph which formed from graph K_3 according to definition 2 is in figure 4.

Figure 4. The results of the vertex and side cutset

Through the vertex cutset and side cutset with $k(G) = k'G = k = 2$ the obtained graph is not connected with two components, namely a graph connected to two vertices and an isolated vertex graph. Therefore graph K_3 is2-connected graph. Figure 4 is not the desired graph.

When $n = 4$, according to the definition 1 graph formed is like figure 5.

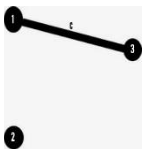


Figure 5. K_4 graph

Since Figure 5 satisfies Theorem 1, Figure 5 fulfills the initial stage and forms graph K_4. After that, the next step is carried out. According to definition 2, we get a k-connected graph from K_4 graph as shown in Figure 6.

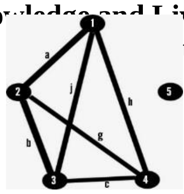


Figure 6. The results of the vertex and side cutset

According to Figure 6, the results of the vertex and side cutsets are $k(G) = k'G = k = 3$. Because it satisfies theorem 2 and theorem 3, then graph K_4 is a 3-connected graph and is not the desired graph.

When $n = 5$, the graph representation according to definition 1 is like Figure 7.

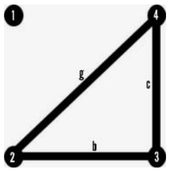


Figure 7. K_5 graph

Since Figure 7 satisfies Theorem 1, figure 7 is a graph K_5 and fulfills the initial stage. Next is to form grapg K_5 into a k-connected graph. According to definition 2, the image formed is in figure 8.

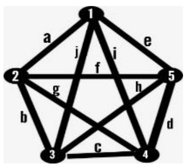


Figure 8. The results of the vertex and side cutset

According to figure 8, the result of the vertex and edge cutset is $k(G) = k'G = k = 4$.Because it fulfills theorem 2 and theorem 3, then graph K_5 is a 4-connected graph and is not desired graph.

When $n = 6$, the graph representation according to definition 1 is like figure 9.

Figure 9. Graph K_6

Because the figure 9 satisfies theorem 1, then figure 9 fulfills the initial stage and forms graph K_6. Next form a k-connected graph from graph K_6 according to definition 2. We get a k-connected shown in figure 10.

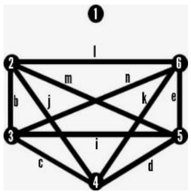


Figure 10. The results of the vertex and side cutset

According to figure 10, the result of the vertex and edge cutset is $k(G) = k'G = k = 5$. Because it satisfies theorem 2 and theorem 3, then graf K_6 is a 5-connected graph and is the desired graph.

B. *Hamiltonian* in *5-connected graph*

After finding the 5-connected graph is the desired graph, it will be proven wether *5*-connected graph is Hamiltonian.

Definition 3

A connected graph is called Hamiltonian if it contains Hamiltonian circuits. The circuit in the Hamiltonian visits every vertex of the graph exactly once except for the initial vertex because $v_0 = v_n$.

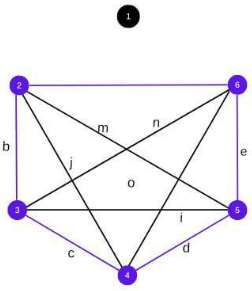
Definition 4

Uniquely Hamiltonian is a graph with a single Hamiltonian circuit.

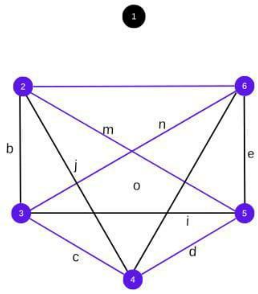
Theorem 4

In the complete graph where $n \geq 3$, contains as many Hamilton circuits as $\frac{1}{2}(n-1) \geq 1$.

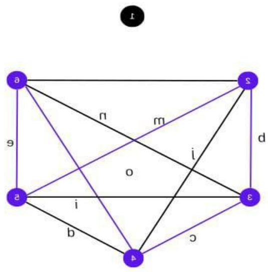
Based on theorem 4, it will be proved the number of Hamiltonian circuits in a 5-connected graph. The Hamilton 5-connected graph circuit can be seen in Figure 11.



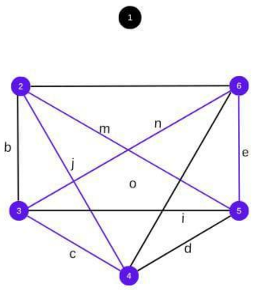
Circuit 1



Circuit 2



Circuit 3



Circuit 4

Figure 11. Circuit Hamilton 5-connected graph

According to theorem 4, Hamilton circuits in a 5-connected graph are four. This proves that the 5-connected graph meets the definition of 3 in other words the 5-connected graph is a Hamiltonian. According to the definition of 4, the 5-connected graph is not a Hamiltonian because it has four Hamilton circuits, which means that the Hamilton circuit is not single.

Based on the above results, the k-connected graph can be applied to real life, one example is communication networks. If the graph G is connected, then any two or more computers in the network can be connected or communicate with each other. However, it is still possible for the network to be connected if there is a disconnected connection. This can be analyzed through the connectedness of a graph. The smaller the vertex and edge cutset results, the weaker the connection. Vice versa, the greater the vertex and edge cutset, the stronger the connectedness of a graph.

IV. CONCLUSION

Based on the results and discussion in the previous chapter, the evidence obtained from the 5-connected graph is as follows:

a. The 5-connected graph is a form of k-connected graph of a complete six-vertex graph or graph K_6.
b. The 5-connected graph is Hamiltonian because it has a Hamiltonian circuit.
c. The 5-connected graph is not uniquely Hamiltonian because it has four Hamiltonian circuits.

In this article, the author focuses on the discussion of Hamiltonian and 5-connected graph. Based on the results obtained, the authors hope that further research will focus on discussing the uniquely Hamiltonian and its application.

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