Dirac Equation for Posch-Teller Potential in Radial Section Symmetry Spin Case using Asymptotic Iteration Method

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Abstract— This study aims to determine the value of the energy spectrum and wave function for the Posch-Teller potential in the case of radial spin symmetry. The solution to the Dirac equation using the asymptotic iteration method is done by reducing the second-order differential equation to a hypergeometric type differential equation by means of variable substitution to obtain a relativistic energy equation. The relativistic energy of the system is calculated using matlab software. This study is limited to the case of spin symmetry in the radial section.

Keywords—Dirac Equation; Potential Posch-Teller; Asymptotic Iteration Method

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I. INTRODUCTION

In 1928 the British physicist Paul Dirac investigated the relativistic covariance wave equation known as the Dirac equation(Y. Alam, Suparmi, Cari, & Anwar, 2016)(Yuniar Alam, 2015). The Dirac equation describes the corresponding basis for the spin elemental particle ½ for an electron. The Dirac equation is consistent with both the principles of quantum mechanics and the special theory of relativity(Suparmi, 2012). Direct solution of the Dirac equation of a system of particles by determining the energy(Meyur, 2011)(Pramono, Suparmi, & Cari, 2016) and wave function(Suparmi, 2013)(Guzmán Adán, Orelma, & Sommen, 2019) of a particle affected by a potential(Y. Alam et al., 2016) whose potential energy is a function of position(S. M. Ikhdair, Hamzavi, & Rajabi, 2013). The solution to the Dirac(Chen, 2019) equation can be solved by reducing the Dirac equation to a Second Order Differential equation.

There are several types of potential in quantum(Alvarez-Castillo, 2008) to describe particle dynamics in quantum mechanics. Some examples of these potentials are the Coloumb, Morse, Rosen-Morse, Manning Rosen, Poschl-Teller group, Gendensthein group, Symmetrical Top, Eckart, Scraft and Kepler group in the hypersphere. This paper uses a different method from previous research. This paper presents the solution of the Dirac equation for the q-deformed Posch-Teller potential in the case of radial spin symmetry(Salvat & Fernández-Varea, 2019) using the asymptotic iteration method.

II. RESEARCH METHOD

Asymptotic Iteration Method

The asymptotic iteration method(Husein, 2014) is the method used to obtain an exact solution of a second order linear homogeneous differential:

$$y_n''(x) - \lambda_o(x) y_n'(x) + s_o(x) y_n(x) = 0 \qquad (1)$$

Where $\lambda_o \neq 0$ and s_o the first derivative show the relationship with x, another parameter i.e. n is defined as a radial quantum number. To obtain a general solution to this equation, differentiating Equation (1), which depends on x, is obtained

$$y_n''' = \lambda_1(x) + s_1(x) y_n(x) \qquad (2)$$

If defined

$$\lambda_k(x) = \lambda_{k-1}'(x) + s_{k-1}(x) + \lambda_{k-1}\lambda_o(x) \qquad (3)$$

$$s_k(x) = s_{k-1}'(x) + s_o(x)\lambda_{k-1}(x) \qquad (4)$$

$$k = 1,2,3,..... \qquad (5)$$

$$\lambda_k(x)s_{k-1}(x)-\lambda_{k-1}(x)s_k(x)=0=\Delta_k,\quad k=1,2,3,\ldots \tag{6}$$

with and is a function of c∞ (coefficient of differential equation). The asymptotic iteration method(Pratiwi, Suparmi, Cari, & Husein, 2017) is applied directly to some problems if a wave function is known in advance and satisfies the boundary conditions of zero (0) and infinity point (∞). Equation (1) can be easily iterated until (*k+1*) and (*k+2*)(Andrade, Silva, Ferreira, & Rodrigues, 2014), *k=1,2,3,*. . . so that obtained

$$y_n^{k+1}(x)=\lambda_{k-1}'(x)y_n'(x)+s_{k-1}(x)y_n(x) \tag{7}$$

$$y_n^{k+2}(x)=\lambda_k'(x)y_n'(x)+s_k(x)y_n(x) \tag{8}$$

With

$$\lambda_k(x)=\lambda_{k-1}'(x)+s_{k-1}(x)+\lambda_o(x)\lambda_{k-1}(x) \tag{9}$$

$$s_k(x)=s_{k-1}'(x)+s_o(x)s_{k-1}(x) \tag{10}$$

From equation (2) obtained the relationship:

$$\frac{y_n^{(k+2)}(z)}{y_n^{(k+1)}(z)}=\frac{\lambda_k(z)\left[y_n'+\frac{s_k(z)}{\lambda_k(z)}f(z)\right]}{\lambda_{k-1}(z)\left[y_n'+\frac{s_{k-1}(z)}{\lambda_{k-1}(z)}f(z)\right]} \tag{11}$$

For *k* large enough, if

$$\frac{s_k(z)}{\lambda_k(z)}=\frac{s_{k-1}(z)}{\lambda_{k-1}(z)}=\alpha(z) \tag{12}$$

While the eigenfunctions for equation (1) can be solved using(Soylu, Bayrak, & Boztosun, 2008)

$$y_n(x)=C'e^{-\int\alpha_n(x)dx} \tag{13}$$

Another form of equation (1) is written as follows

$$y''(x)=2\left(\frac{tx^{N+1}}{1-bx^{N+2}}-\frac{c+1}{x}\right)y'(x)-\frac{wx^n}{1-bx^{N+2}} \tag{14}$$

Equation (14) is an AIM type differential solution that will be used to determine the wave function equation of the Dirac equation. Equation (14) can be solved by using the equation:

$$y_n(x)=(-1)^n C'(N+2)^n(\sigma)_{n2}F_1\left(-n,p+n,\sigma,bx^{N+2}\right) \tag{15}$$

With

$$(\sigma)_n=\frac{\Gamma(\sigma+n)}{\Gamma(\sigma)} \tag{16}$$

$$\sigma=\frac{2c+N+3}{N+2} \tag{17}$$

and

$$p = \frac{(2c+1)b + 2t}{(N+2)b} \tag{18}$$

Here C' is the radial normalization constant and dan $_2F_1$ is a hypergeometric function.

III. RESULT AND DISCUSSION

The relativistic Schrodinger equation is referred to as the Gordon Client equation for integer spins and the Dirac equation for spin . Quantitative description of the relativistic(S. M. Ikhdair & Hamzavi, 2012) particle motion which is affected by the field force which is represented as the potential energy of the spinning particle(Salvat & Fernández-Varea, 2019) is expressed in the form of a differential equation called the Dirac equation. The Dirac equation for vector potential V(r) and scalar S(r) is written as follows:

$$\{c\vec{\alpha}.\vec{p} + \beta(Mc^2 + s(r))\}\psi(r) = \{E - v(r)\}\psi(r) \tag{19}$$

With the relativistic mass M of the particle, E the total energy, and the linear momentum operator, the values of and are expressed in terms of the equation

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \tag{20}$$

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{21}$$

With $\vec{\alpha}$ Pauli I matrix 2x2 identitas identity matrix

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{22}$$

The spin dirac can be written as follows:

$$\psi(r) = \begin{pmatrix} f_{nk}(r) \\ g_{nk}(r) \end{pmatrix} = \begin{pmatrix} \dfrac{F_{nk}(r)}{r} Y_{jm}^{l}(\theta,\varphi) \\ i\dfrac{G_{nk}(r)}{r} Y_{jm}^{\bar{l}}(\theta,\varphi) \end{pmatrix} \tag{23}$$

With $F_{nk}(r)$ is the dirac upper spin(Sari, Suparmi, & Cari, 2015) component and $G_{nk}(r)$ is the lower Dirac pseudospin(S. Ikhdair & Sever, 2007) component. $Y_{jm}^{l}(\theta,\varphi)$ is the spherical harmonic spin(Ding & Liu, 2015)(Potential, 1929), l is the orbital quantum number and l is the orbital pseudospin quantum number, m is the projection of the angular momentum(Taşkın, 2009)(S. M. Ikhdair & Hamzavi, 2013) on the z-axis. By substituting equation (23) into equation (19), we get

$$\left(\frac{d}{dr}+\frac{k}{r}\right)F_{nk}(r)= \left(E-V(\vec{r})+Mc^2+S(\vec{r})\right)G_{nk}(r) \tag{24}$$

$$\left(\frac{d}{dr}-\frac{k}{r}\right)G_{nk}(r)= \left(E-V(\vec{r})-Mc^2-S(\vec{r})\right)F_{nk}(r) \tag{25}$$

After elimination $F_{nk}(r)$ and $G_{nk}(r)$ from equation (24) and equation (25), two differential equations similar to the Schrodinger(Ikot, Awoga, & Antia, 2013)(S. M. Ikhdair & Sever, 2007) equation for components $F_{nk}(r)$ and components $G_{nk}(r)$ are obtained.

$$\left(\frac{d^2}{dr^2}-\frac{k(k+1)}{r^2}\right)F_{nk}(r)+\left(E+Mc^2\right) \left(\frac{d}{dr}+\frac{k}{r}\right)F_{nk}(r)= \left[\left(E+Mc^2\right)\left(E-Mc^2\right)\right]F_{nk}(r) \tag{26}$$

$$\left(\frac{d^2}{dr^2}-\frac{k(k-1)}{r^2}\right)G_{nk}(r)+\left(Mc^2+E\right) \left(\frac{d}{dr}+\frac{k}{r}\right)G_{nk}(r)= \left[\left(E-2V(\vec{r})-Mc^2\right)\left(E-Mc^2\right)\right]G_{kr}(r) \tag{27}$$

With $S(\vec{r})=V(\vec{r})$ (28)

Dirac equation for the case of spin symmetry $c=\hbar=1$ so that the equation becomes

$$\left[\frac{d^2}{dr^2}-\frac{k(k+1)}{r^2}\left(E+M\right)\left(E^2-M^2\right)\right]F_{nk}(r)=0 \tag{29}$$

If $S(\vec{r})=V(\vec{r})$ is the q-deformed Poschl-Teller potential(Deta & Suparmi, 2015), which is defined as

$$V_{PT}(r)=\alpha^2\left(\frac{v_o}{\sinh_q^2\alpha r}+\frac{v_1}{\cosh_q^2\alpha r}\right) \tag{30}$$

$$\left(\frac{d^2}{dr^2}-[E+M]\alpha^2\left(\frac{v_o}{\sinh_q^2\alpha r}+\frac{v_1}{\cosh_q^2\alpha r}\right)+\left[E^2-M^2\right]-l(l+1)\right)\frac{F_{nk}(r)}{r^2}=0 \tag{31}$$

By substituting the variables $\cosh_q^2\alpha r = z$ in equation (31) and also for example $\frac{1}{r^2}\cong\frac{\alpha^2}{\sinh_q^2\alpha r}$, then we get the form of

$$z(q-z)\frac{d^2F_{nk}(r)}{dr^2}+\frac{1}{2}(q-2z)\frac{dF_{nk}(r)}{dr}-\left\{[E+M]\left(\frac{v_o}{4(q-z)}+\frac{v_1}{4z}\right)-\frac{\left[E^2-M^2\right]}{4\alpha^2}+\frac{l(l+1)}{4(q-z)}\right\}F_{nk}(r)=0 \tag{32}$$

Furthermore, equation (32) is reduced to an equation of the hypergeometric type through the example of a wave function.

$$F_{nk}(z)=z^{\delta}(q-z)^{\gamma}f \tag{33}$$

After manipulating equations (32) and (33), we get

$$f''=\left(\frac{(2\delta+2\gamma+1)z-\left(2\delta+\frac{1}{2}\right)q}{z(q-z)}\right)f'+\left(\frac{\frac{\left[E^2-M^2\right]}{4\alpha^2}+(\delta+\gamma)^2}{z(q-z)}\right)f \tag{34}$$

Equation (34) is a second order equation. By comparing equation (34) with equation (1), we can write λ_o and s_o, then we can calculate λ_k and s_k .

$$\lambda_o=\frac{\left((2\delta+2\gamma+1)z-\left(2\delta+\frac{1}{2}\right)q\right)}{z(q-z)}$$

$$s_0=\frac{\left((\delta+\gamma)^2+\frac{\left[E^2-M^2\right]}{4\alpha^2}\right)}{z(q-z)}$$

$$\lambda_1=\left[\frac{\left(2\delta+\frac{1}{2}\right)}{z^2}+\frac{\left(2\gamma+\frac{1}{2}\right)}{(q-z)^2}\right]+\left[-\frac{\left(2\delta+\frac{1}{2}\right)}{z}+\frac{\left(2\gamma+\frac{1}{2}\right)}{(q-z)}\right]^2+\left[\frac{C}{z}+\frac{C}{(q-z)}\right]$$

$$s_1=\left[\frac{C}{z^2}+\frac{C}{(q-z)^2}\right]+\left[\frac{C}{z}+\frac{C}{(q-z)}\right]\left[-\frac{\left(2\delta+\frac{1}{2}\right)}{z}+\frac{\left(2\gamma+\frac{1}{2}\right)}{(q-z)}\right]$$

By combining the above results with equation (6), we get

$$\Delta_o = s_o\lambda_1 - s_1\lambda_o = 0 \rightarrow \varepsilon_o = (\delta+\gamma)^2$$
$$\Delta_1 = s_1\lambda_2 - s_2\lambda_1 = 0 \rightarrow \varepsilon_1 = (\delta+\gamma+1)^2 \qquad (35)$$
$$\Delta_2 = s_2\lambda_3 - s_3\lambda_2 = 0 \rightarrow \varepsilon_2 = (\delta+\gamma+2)^2$$

And so on, with $\varepsilon_r = (M^2 - E^2)\frac{1}{4\alpha^2}$ From equation (35), it can be regenerated to

$$\varepsilon = (\delta + \gamma + n_r)^2 \qquad (36)$$

With n_r is a radial quantum number, so the eigen energies are

$$\varepsilon = \left(\frac{1}{2}q\sqrt{[E+M]v_1 + \frac{1}{4}q} \pm \frac{1}{2}q\sqrt{[E+M]v_o + l(l+1) + \frac{1}{4}q} + n_r + \frac{1}{2}q^2\right)^2 \qquad (37)$$

$$(M^2 - E^2)\frac{1}{4\alpha^2} = \left(\frac{1}{2}q\sqrt{[E+M]v_1 + \frac{1}{4}q} \pm \frac{1}{2}q\sqrt{[E+M]v_o + l(l+1) + \frac{1}{4}q} + n_r + \frac{1}{2}q^2\right)^2 \qquad (38)$$

As for the function f $f = (-1)^n C'(1)^{n_r}\left(2\delta + \frac{1}{2}\right)_{nr} {}_2F_1\left(-n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, q\right)$ (39)

So that the total radial function can be written $F = Z^\delta (q-z)^\gamma (-1)^n C'(1)^{n_r}\left(2\delta + \frac{1}{2}\right)_{nr} {}_2F_1\left(-n_r, 2\delta + 2\gamma + n_r, 2\delta + \frac{1}{2}, q\right)$

(40)

The energy results obtained for the case of spin symmetry can be seen in Table 1.

Table 1. Poschl-Teller potential energy spectrum for the symmetrical spin case with $M = 5; v_1 = 1; v_2 = 0{,}75; \alpha = 0{,}2.$

Untuk k=l			E_{nk}
l	n	k	
0	0	0	4.66350
1	0	1	4.61978
2	0	2	4.53329
3	0	3	4.40430
4	0	4	4.23110
0	1	0	4.50477
1	1	1	4.44916
2	1	2	4.34050
3	1	3	4.18046
4	1	4	3.96748
0	2	0	4.30662
1	2	1	4.23756
2	2	2	4.10344
3	2	3	3.90697
4	2	4	3.64569
0	3	0	4.06434
1	3	1	3.97946
2	3	2	3.81493
3	3	3	3.57365
4	3	4	3.25007

The energy in table 1 can be graphed as shown in Figure 1.

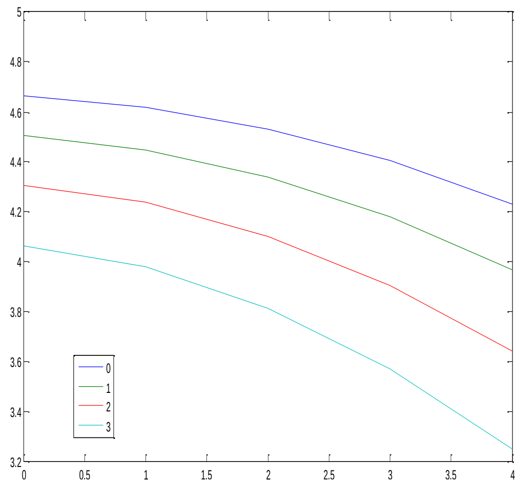

Figure 1. The results of the energy spectrum calculation for the symmetry case at a certain nr.

IV. CONCLUSION

The solution of the Dirac equation for the q-deformed Posch-Teller potential in the case of spin symmetry in the radial section is done by reducing the second-order differential equation into a differential equation of hypergeometric type through variable substitution and the appropriate wave function. By comparing the second-order differential equation and the hypergeometric type with the second-order linear homogeneous differential equation for the asymptotic iteration method, the relativistic energy equation and weight function are obtained. The relativistic wave function is obtained from the weight function and expressed in the form of asymptotic iterations. The relativistic energy spectrum of the system was calculated using the Matlap software.

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